Cosmological Black Holes

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Abstract

In this letter we propose the existence of low density black holes and discuss its compatibility with the cosmological observations. The origin of these black holes can be traced back to the collapse of long wavelength cosmological perturbations during the matter dominated era, when the densities are low enough to neglect any internal and thermal pressure. By introducing a threshold density $\hat{\rho}$ above which pressure and non-gravitational interactions become effective, we find the highest wavelength for the perturbations that can reach an equilibrium state instead of collapsing to a black hole. The low density black holes introduced here, if they exist, can be observed through weak and strong gravitational lensing effects. Finally we observe that we obtained here a cosmological model which is capable to explain in a qualitative way the void formation together with the value $\Omega = 1$. But we remark that it needs to be improved by considering non spherical symmetric black holes.

Even if the concept of black hole has been put forward more than 200 years ago[1],it is only in general relativity that a self consistent definition of black holes has been given as a consequence of the Schwarzschild solution.

Later the concept of black hole has been extended to arbitrary spacetimes, and it is generally defined as a connected region of a space-like surface from which particles and photons cannot escape to the future null infinity[7].

In the following we shall consider only Schwarzschild black holes, which are defined as bodies of mass M lying completely within a sphere of radius

$$R_s = \frac{2GM}{c^2}. \tag{1}$$

called the Schwarzschild radius[8].

The pioneering work of Chandrasekhar[2] about the limiting mass value for white dwarfs and Oppenheimer and Snyder[3] for the collapse of a uniform density body with vanishing pressure, lead to consider the existence of astrophysical black holes as the ending stage of massive stars. Later it was also supposed that primordial black holes could have formed in the early universe as a results of density perturbations [4][5][6]. In both cases black holes have been considered as very high density objects formed at very extreme conditions. In this letter we want to stress that this assumption is not necessarily true. For this purpose let us first re-express the Schwarzschild radius in terms of the mass-energy density ϵ. Given a spherical body of mass M and radius R it is straightforward to see that

$$R_s = \sqrt{\frac{3c^2}{8\pi G\rho\alpha^3}}. \tag{2}$$

with

$$\alpha = \frac{R}{R_s}$$

and $\rho = \epsilon/c^2$.

The body is a black hole if $\alpha \leq 1$. Taking $\alpha = 1$, we can define the function[9]

$$R_s(\rho) = \sqrt{\frac{3c^2}{8\pi G\rho}}, \tag{3}$$

which associates to any density a corresponding Schwarzschild radius; it means that in a medium of uniform density ρ, any space-like sphere with radius, at least, equal to $R_s(\rho)$ is a black hole. It is evident here that the density of a black hole is an arbitrary quantity.

It is remarkable that, even if the relation (2) appears to be very simple and natural, it has been generally overlooked in most of the literature. In this letter we want to show that it yields some new results.

Equation (3) leads to consider the black holes for any value of ρ. Together with the astrophysical and primordial black holes [10], one can consider the existence of black holes with very low densities. As they are very massive, we shall discuss their existence compatibly with the astrophysical and cosmological observations.

Let us suppose to have in a homogeneous and isotropic universe with zero-pressure a distribution of spherical voids of fixed comoving radius R_v and that each void has been obtained by evacuating a spherical region and compensating the evacuated rest mass by a black hole.

From the Birkhoff theorem we know that this operation does not affect the spacetime outside the void[11].

The black hole mass M is given by

$$M = \frac{4}{3}\pi\Omega_{cbh}\rho_{crit.}R_v^3, \tag{4}$$

where

$$\Omega_{cbh} = \frac{\rho_{cbh}}{\rho_{crit.}} \tag{5}$$

represents the contribution of all these black holes to the total density of the universe.

The picture described here represents very schematically, the actual distribution of matter as has been detected over the last twenty years, since a 60 Mpc large void in the Böotes constellation was discovered[12]).

Ever since systematic surveys have shown the existence of many regions with similar characteristics. Complete magnitude limited surveys ([13], [14] and [15]) have shown that galaxies tend to lie on sheet-like structures surrounding voids with a typical size of about $40-50h^{-1}Mpc$ [16][21] [22], but which can vary from $20h^{-1}Mpc$ to $100h^{-1}Mpc$ of diameter. We note that the shape and the dimensions of the voids depend upon how they are defined (see[16], [17]).

Generally two models of universe have been used to describe these voids, the Swiss Cheese model and the Tolman-Bondi model(for a review of both models see [26]). In this letter we shall adopt a different point of view.

In the following we shall make two assumptions,

a) there is in all the voids a central black hole with mass M given by (4). We are making the hypothesis that a large amount of dark matter can be thought as matter hidden inside these large black holes; the value of Ω_{cbh} can be chosen to match with the recent observational results of Boomerang[18] which are consistent with a flat universe;

b) we will consider all the voids spherical, as a first approximation to their real shape, which is generally ellipsoidal.

We determine for each void the corresponding mass, the Schwarzschild radius from equations (3) and (4) and consequently the mass-energy density of the central black hole. The results are listed in Table 1.

TABLE 1

Void diameter	Mass/$\Omega_{cbh}h^{-1}$	$R_s/\Omega_{cbh}h^{-1}$	density
$30h^{-1}Mpc$	$3.9 \times 10^{15}M_\odot$	$0.37Kpc$	$5.02 \times 10^{-15}g/cm^3$
$50h^{-1}Mpc$	$1.8 \times 10^{16}M_\odot$	$1.7Kpc$	$2.34 \times 10^{-16}g/cm^3$
$100h^{-1}Mpc$	$1.4 \times 10^{17}M_\odot$	$13.9Kpc$	$3.66 \times 10^{-18}g/cm^3$

It is noteworthy that the highest densities involved are of the order of $10^{-15}g/cm^3$. This confirms the hypothesis that the black holes considered have very low densities.

We are able to formulate an elementary model to explain the formation of these "cosmological black holes" (CBH).

Usually the voids are considered as the result of the evolution of negative perturbations, which produced underdense regions [22].

Here we propose, instead, that these voids were generated by the collapse of long wavelength cosmological perturbations to a black hole during the matter dominated era.

According to the inflationary scenario, we only need to suppose that the inflation occurred during a time long enough to provide such perturbations.

A growing perturbation in a homogeneous and isotropic universe, with initial wavelength λ_i and amplitude $\delta_i << 1$, evolves according to the equation

$$\left(\frac{\dot{a}_p}{a_i}\right)^2 = H_i^2\left(\Omega_p(t_i)\frac{a_i}{a_p} + 1 - \Omega_p(t_i)\right), \tag{6}$$

where a_i is the expansion factor at the beginning of the perturbation formation and Ω_p is the ratio of the perturbation density and the background critical density

$$\Omega_p(t_i) = \frac{\rho(t_i)(1+\delta_i)}{\rho_c(t_i)} = \Omega(t_i)(1+\delta_i). \tag{7}$$

The solution of equation (6) together with the relation

$$\lambda(t) = \lambda_i\frac{a(t)}{a_i} \tag{8}$$

give the evolution of $\lambda(t)$ in the parametric form

$$\lambda(\theta) = \frac{\lambda_i}{2}\frac{\Omega_p}{\Omega_p - 1}(1-\cos\theta) \tag{9}$$

and

$$t(\theta) = \frac{1}{2H_i}\frac{\Omega_p}{(\Omega_p - 1)^{\frac{3}{2}}}(\theta - \sin\theta). \tag{10}$$

Let us assume that the low densities given in the Table 1 represent respectively the final ones of the whole process. This suggests, that the perturbations undergo a smooth transition[19] to a black hole when their density and $\lambda(t)$ satisfy the relationship (3), regardless of whether they have entered in a non-linear regime.

We stress here that this is a possible as the mass-energy densities considered are always low enough to neglect any contribution of the internal pressure and any other phenomena which could prevent the black hole formation.

On the other side it is important to note that, using a sentence in [20], the collapsing perturbation “behaves as if there existed no cosmic expansion or curvature”.

We are in the physical situation described by the collapse model of Oppenheimer and Snyder[3], where the total mass of the perturbation

$$M = \frac{\pi}{6}\Omega_p \rho_i \lambda_i^3 \tag{11}$$

is constant during the expansion and contraction of the perturbation.

Accordingly, the Schwarzschild radius for the spherical perturbation is

$$R_s = \frac{H_i^2}{c^2}\left(\frac{\lambda_i}{2}\right)^3 \Omega_p, \tag{12}$$

We can distinguish two cases.

First

$$\frac{2R_s}{\lambda} \geq 1, \tag{13}$$

implies that the perturbation becomes a black hole when it crosses the Hubble horizon if

$$\left(\frac{\lambda_i}{2}\right)^2 \geq \frac{c^2}{H_i^2 \Omega_p}. \tag{14}$$

In the second case

$$\frac{2R_s}{\lambda_i} < 1, \tag{15}$$

the perturbation evolves according to equations (9) and (10). We expect generally that, after reaching the maximum expansion at $\theta = \pi$ it collapses to a black hole.

Now let us establish the conditions for a perturbation *not to become* a black hole. For this purpose, we fix a threshold value $\hat{\rho}$ of the mass-energy density, above which the equation of state changes, the internal pressure cannot be neglected and non-gravitational phenomena arise and equilibrium conditions can be reached, preventing any further collapse[9].

To this density there corresponds a value of $\lambda = \hat{\lambda}$ defined by the equation

$$M = \frac{\pi}{6}\hat{\rho}\hat{\lambda}^3. \tag{16}$$

Comparing it with equation (11), we obtain

$$\hat{\lambda}^3 = \frac{\rho_i \Omega_p}{\hat{\rho}} \lambda_i^3. \tag{17}$$

The perturbation does *not collapse* to a black hole if $\hat{\lambda} > R_s$, i.e. if

$$\left(\frac{\lambda_i}{2}\right)^2 < \frac{3c^2}{8\pi G} \frac{1}{(\rho_i \Omega_p)} \left(\frac{\rho_i \Omega_p}{\hat{\rho}}\right)^{\frac{1}{3}}. \tag{18}$$

In conclusion, among all wavelengths of the cosmological perturbations spectrum, only those structures which satisfy (18) appear in the observed universe as *luminous* matter or *exotic* dark matter. The rest of the matter is confined in very massive cosmological black holes.

After the collapse of the perturbation, the region around it expands in a comoving way leading to the formation of a void region between it and the rest of the universe. The central black hole cannot be seen, the entire region appears to an external observer as a void.

Usually all kinds of dark matter can be detected only through their gravitational lensing properties. In the following we will evaluate the orders of degree corresponding to these cosmological black holes.

A black hole in the center of a void acts as a perfect Schwarzschild lens [23] and [24]. Regarding the weak gravitational lensing, we expect for a 50 Mpc void a large angular scale given by

$$\alpha_0 \approx 1.2 \times 10^{-6} \sqrt{\frac{M}{M_\odot}} \sqrt{\frac{h_{50}}{\eta'}} \sim 10^2 arcsec, \tag{19}$$

and the corresponding length scale is

$$\xi_0 \approx 0.03 \sqrt{\frac{M}{M_\odot}} \sqrt{\frac{\eta}{h_{50}}} \sim 3\text{Mpc}, \tag{20}$$

where η' and η are numbers of the order of unity obtained by comparing the distances of the source and the CBH from the observer with the present Hubble radius (see discussion in [23]).

The high values of the masses involved lead also to expect strong gravitational lensing effects at distances of about $3R_s$[24], which in the case of a 50 Mpc void $\sim$ 6 Kpc. As a consequence there is the formation of a sequence of *relativistic images*. These are, theoretically, an infinite sequence of images on both sides of the optical axis.

A complete discussion of the lensing properties of the CBHs is in preparation [27].

To conclude this work we point out first that the idea that black holes can be generated by cosmological perturbations is not new[4][5][6][10]. But the role of primordial black holes as dark matter candidates is limited by very severe constraints[28].

There is even some evidence of supermassive black holes residing at the centers of the galaxies with masses of the order of $10^9 M_\odot$ [25].

We have proposed in this letter the existence of a different kind of black holes with larger masses and isolated with respect to the galaxy distributions. If they exist, probably they were originated directly by the collapse of cosmological perturbations with large initial wavelengths during the matter dominated era.

The universe model that comes out from this picture resembles very much with already known cosmological models. First of all, it reminds the Lindquist-Wheeler model[29], which was introduced to study numerically the dynamics of the universe. But it is also very similar to the Swiss Cheese inhomogeneous model[26], where the typical vacuoles have been substituted by the large mass CBHs within a spherical void.

It must be noted that the existence of the cosmological black holes is compatible with the Zeldovic perturbation spectrum and with any other spectrum. The only information on the spectrum has to be deduced from the void distribution which reflects the initial perturbation distribution before the black hole formation.

A good feature of this model is that it is testable. According to the catalogs of the voids (see e.g.[16]) the CBHs must be very few, so it is not an easy task to locate them. But on the other side they have very interesting lensing properties in the weak gravity lensing and in the strong gravity lensing regimes.

The resulting cosmological model gives a good qualitative description of the present universe, but we think that it needs to be refined further on. A more realistic approach should be to consider perturbations which are not spherical. In this case the Birkhoff theorem cannot be invoked and the results of Einstein and Straus[20] no longer hold. Then one should consider the effect of the universe expansion against the collapse of the perturbations, which may influence the threshold density $\hat{\rho}$. If we suppose that for an isolated body one can take $\hat{\rho} \sim 10^{14} g/cm^3$ the nuclear matter density. When there is an interplay between cosmological expansion and the perturbation collapse, one may expect lower values of $\hat{\rho}$. In this more general context, we should expect a large contribution of the cosmological black holes to the gravitational waves background.

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